Published by Institute of Physics Publishing for SISSA/ISAS
Received: *March 5, 2003*
Accepted: *March 11, 2003*

Supersymmetric domain walls and strings in $D = 5$ gauged supergravity coupled to vector multiplets

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Abstract: We present new supersymmetric domain wall and string solutions of five-dimensional $N = 2$ gauged supergravity coupled to an arbitrary number of vector multiplets. Using the techniques of very special geometry allows to obtain the most general domain wall preserving half of the supersymmetries. This solution, which describes a renormalization group flow in the dual field theory, is given in terms of Weierstrass elliptic functions. The magnetically charged, one quarter supersymmetric string solutions are shown to be closely related to Liouville theory. We furthermore investigate general product space compactifications, and show that topological transitions from $AdS_3 \times S^2$ to $AdS_3 \times H^2$ can occur when one moves in moduli space.

Keywords: AdS-CFT and dS-CFT Correspondence, Supergravity Models.

http://jhep.sissa.it/archive/papers/jhep032003023/jhep032003023.pdf

JHEP03(2003)023

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1. Introduction

The conjectured equivalence between string theory on anti-de Sitter (AdS) spaces (times some compact manifold) and certain superconformal gauge theories living on the boundary of AdS [1] has led to an increasing interest in solutions of gauged supergravities that preserve some fraction of supersymmetry. On the CFT side, these supergravity vacua could correspond to an expansion around non-zero vacuum expectation values of certain operators, or describe a holographic renormalization group flow [2]. In that way, one can study strongly coupled field theories using classical supergravity solutions. Of particular interest in this context are solutions that describe black holes [3]–[5], domain walls [6] or strings [7]–[9].

In our paper we will concentrate on five-dimensional $N = 2$ gauged supergravity coupled to vector multiplets, which is relevant for holographic descriptions of field theories in four dimensions with less than maximal supersymmetry. In particular, we give a systematic treatment of supersymmetric domain walls and strings. Using the tools of very special geometry, which underlies the considered supergravity theory, allows to obtain the most general domain wall preserving half of the supersymmetries. This solution, which describes a holographic renormalization group flow, turns out to be given in terms of the Weierstrass elliptic function. We show that the RG flow is a so-called vev flow, and derive

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the holographic beta and $\mathcal{C}$-functions. The geometry has a curvature singularity signaling nontrivial IR physics. Applying the existing criteria for allowed singularities we obtain conditions that the invariants of the elliptic function must satisfy.

Magnetically charged string-like solutions of gauged supergravities in five dimensions have been derived first in [7, 8]. In [8] it was suggested that solutions interpolating between AdS_5 and $\mathrm{AdS}_3 \times \mathrm{H}^2$ have a holographic interpretation as a four-dimensional CFT that is given a relevant perturbation and flows to a two-dimensional CFT in the IR. This aspect was then elaborated by Maldacena and Nuñez [9]. In the present work, magnetically charged strings preserving one quarter of the supersymmetries will be studied in more detail. In particular, we show that for the case of constant scalar fields the solutions are given in terms of functions that satisfy the Liouville equation. They can thus be classified according to their monodromy, which can be elliptic, hyperbolic or parabolic. The monodromy is determined by the Liouville momentum, which turns out to be related to the curvature of the two-dimensional Riemann surfaces into which the space transverse to the strings is sliced.

We furthermore investigate general product space compactifications of the five-dimensional theory. These geometries preserve half of the supersymmetries and are of the form $\mathrm{AdS}_3 \times \mathrm{S}^2$ or $\mathrm{AdS}_3 \times \mathrm{H}^2$. It will be shown that topological transitions from the former to the latter case can appear when one moves in moduli space. In certain cases the product geometries can arise as near-horizon limit of the one quarter supersymmetric strings mentioned above. One has then supersymmetry enhancement near the horizon.

The remainder of this paper is organized as follows: In order to make the paper self-contained, we briefly review $D = 5$, $N = 2$ gauged supergravity coupled to vector multiplets in section 2. In section 3 we write down a general ansatz for supersymmetric solutions that includes both string-like and domain wall configurations. The Killing spinor equations are analyzed and a flow equation for the scalar fields is obtained. In section 4 we derive the most general half supersymmetric domain wall solution and discuss its holographic interpretation. In the following section, the fixed points of the flow equation are studied, and it is shown that they lead either to magnetically charged strings with constant scalars or to product space geometries. We discuss the connection of the magnetic strings with Liouville theory, and show the possibility of topological transitions for the product space "compactifications". In section 6 a new magnetic string solution with varying scalars is presented, and its holographic interpretation is briefly discussed. Electrically charged solutions are considered in section 7. We conclude with some final remarks in section 8.

2. $D = 5$, $N = 2$ gauged supergravity

We start with a brief description of five-dimensional $N = 2$ U(1)-gauged supergravity theories. The fields of these theories consist of a graviton $g_{\mu\nu}$, gravitino ψ_μ, n vector potentials A^I_μ ($I = 1, 2, \ldots, n$), $n-1$ gauginos λ_i and $n-1$ scalars ϕ^i ($i = 1, 2, \ldots, n-1$). The bosonic part of the lagrangian is given by [10]

$$e^{-1}\mathcal{L} = \frac{1}{2}R + g^2 V - \frac{1}{4}G_{IJ}F^I_{\mu\nu}F^{J\mu\nu} - \frac{1}{2}\mathcal{G}_{ij}\partial_\mu\phi^i\partial^\mu\phi^j + \frac{e^{-1}}{48}\epsilon^{\mu\nu\rho\sigma\lambda}C_{IJK}F^I_{\mu\nu}F^J_{\rho\sigma}A^K_\lambda\ , \quad (2.1)$$

where μ, ν are spacetime indices, R is the scalar curvature, $F^I_{\mu\nu}$ denote the abelian field-strength tensors of the vectors A^I_μ, and $e = \sqrt{-g}$ is the determinant of the fünfbein e^a_μ. The scalar potential V is given by

$$V(X) = V_I V_J \left(6X^I X^J - \frac{9}{2}\mathcal{G}^{ij}\partial_i X^I \partial_j X^J \right), \tag{2.2}$$

where V_I are constants, ∂_i denotes a partial derivative with respect to the scalar field ϕ^i and $X^I = X^I(\phi^i)$ are real scalars satisfying the condition $\mathcal{V} = \frac{1}{6}C_{IJK}X^I X^J X^K = 1$. Moreover, G_{IJ} and $\mathcal{G}_{ij}$ can be expressed in terms of the homogeneous cubic polynomial $\mathcal{V}$ which defines a "very special geometry" [11],

$$G_{IJ} = -\frac{1}{2}\frac{\partial}{\partial X^I}\frac{\partial}{\partial X^J}\log \mathcal{V}\Big|_{\mathcal{V}=1}, \qquad \mathcal{G}_{ij} = \partial_i X^I \partial_j X^J G_{IJ}\Big|_{\mathcal{V}=1}.$$

Further useful relations can be found in appendix A. We note that if the five-dimensional theory is obtained by gauging a supergravity theory coming from a Calabi-Yau compactification of M-theory, then $\mathcal{V}$ is the intersection form, X^I and $X_I = \frac{1}{6}C_{IJK}X^J X^K$ correspond to the size of the two- and four-cycles and C_{IJK} are the intersection numbers of the Calabi-Yau threefold.

The supersymmetry transformations of the gravitino ψ_μ and the gauginos λ_i in a bosonic background read [10]

$$\delta\psi_\mu = \left(\mathcal{D}_\mu + \frac{i}{8}X_I({\Gamma_\mu}^{\nu\rho} - 4{\delta_\mu}^\nu\Gamma^\rho)F^I_{\nu\rho} + \frac{1}{2}g\Gamma_\mu X^I V_I \right)\epsilon, \tag{2.3}$$

$$\delta\lambda_i = \left(\frac{3}{8}\Gamma^{\mu\nu}F^I_{\mu\nu}\partial_i X_I - \frac{i}{2}\mathcal{G}_{ij}\Gamma^\mu\partial_\mu\phi^j + \frac{3i}{2}gV_I\partial_i X^I \right)\epsilon, \tag{2.4}$$

where ϵ is the supersymmetry parameter and $\mathcal{D}_\mu$ is the fully gauge and gravitationally covariant derivative

$$\mathcal{D}_\mu\epsilon = \left[\partial_\mu + \frac{1}{4}\omega_{\mu ab}\Gamma^{ab} - \frac{3i}{2}gV_I A^I_\mu \right]\epsilon. \tag{2.5}$$

Here, $\omega_{\mu ab}$ denotes the spin connection and Γ^μ are Dirac matrices.[1]

3. Supersymmetric string solutions

As a general ansatz for supersymmetric string-like solutions we consider metrics of the form

$$ds^2 = e^{2V}(-dt^2 + dz^2) + e^{2W}(dr^2 + r^2 d\Omega_k^2), \tag{3.1}$$

where V and W are functions of the radial coordinate r only,[2] and $d\Omega_k^2$ denotes the standard metric on a two-dimensional surface Σ_k of constant scalar curvature $2k$, where $k = 0, \pm 1$. An explicit form is

$$d\Omega_k^2 = d\theta^2 + F_k^2(\theta)d\phi^2, \tag{3.2}$$

[1]We use the metric $\eta_{ab} = (-,+,+,+,+)$, $\{\Gamma^a, \Gamma^b\} = 2\eta^{ab}$, and $\Gamma^{a_1 a_2 \cdots a_n} = \Gamma^{[a_1}\Gamma^{a_2}\cdots\Gamma^{a_n]}$.

[2]We apologize for using the same symbol V for the scalar potential and for the function appearing in the metric (3.1), but the meaning should be clear from the context.

with

$$F_k(\theta) = \begin{cases} 1\,, & k=0 \\ \sin\theta\,, & k=1 \\ \sinh\theta\,, & k=-1\,. \end{cases} \tag{3.3}$$

Clearly Σ_k is a quotient space of the universal coverings $\mathbb{E}^2$ $(k=0)$, S^2 $(k=1)$ or H^2 $(k=-1)$.

With the choice (3.1), the fünfbein reads

$$e^0_t = e^1_z = e^V\,, \qquad e^2_r = e^W\,, \qquad e^3_\theta = e^W r\,, \qquad e^4_\phi = e^W r F_k\,, \tag{3.4}$$

and the nonvanishing components of the spin connection are given by

$$\begin{aligned}
\omega_t^{02} &= e^{V-W}V'\,,\\
\omega_z^{12} &= e^{V-W}V'\,,\\
\omega_\theta^{23} &= -(W'r+1)\,,\\
\omega_\phi^{24} &= -(W'r+1)F_k\,,\\
\omega_\phi^{34} &= -F_k'\,,
\end{aligned} \tag{3.5}$$

where a prime denotes a derivative.

In five dimensions, strings can carry magnetic charges under the one-form potentials A^I, so we assume that the gauge fields have only a magnetic part, i.e.

$$F^I_{\theta\phi} = kq^I F_k(\theta)\,, \qquad A^I_\phi = kq^I\int F_k(\theta)d\theta\,. \tag{3.6}$$

Note that F^I is essentially the Kähler form on Σ_k.

Plugging the spin connection (3.5) and the magnetic fields (3.6) into the supersymmetry transformations of the gravitino (2.3), we obtain for the Killing spinors ϵ the equations

$$\begin{aligned}
\partial_t\epsilon + \frac{1}{2}e^{V-W}V'\Gamma_{02}\epsilon + \frac{i}{4}kZe^{V-2W}r^{-2}\Gamma_{034}\epsilon + \frac{1}{2}ge^V X^I V_I\Gamma_0\epsilon &= 0\,,\\
\partial_z\epsilon + \frac{1}{2}e^{V-W}V'\Gamma_{12}\epsilon + \frac{i}{4}kZe^{V-2W}r^{-2}\Gamma_{134}\epsilon + \frac{1}{2}ge^V X^I V_I\Gamma_1\epsilon &= 0\,,\\
\partial_r\epsilon + \frac{i}{4}kZe^{-W}r^{-2}\Gamma_{234}\epsilon + \frac{1}{2}ge^W X^I V_I\Gamma_2\epsilon &= 0\,,\\
\partial_\theta\epsilon - \frac{1}{2}(W'r+1)\Gamma_{23}\epsilon - \frac{i}{2}kZe^{-W}r^{-1}\Gamma_4\epsilon + \frac{1}{2}ge^W rX^I V_I\Gamma_3\epsilon &= 0\,,\\
\partial_\phi\epsilon - \frac{1}{2}(W'r+1)F_k\Gamma_{24}\epsilon - \frac{1}{2}F_k'\Gamma_{34}\epsilon + \frac{i}{2}kZe^{-W}r^{-1}F_k\Gamma_3\epsilon + &\\
+\frac{1}{2}ge^W rF_k X^I V_I\Gamma_4\epsilon - \frac{3i}{2}gkV_Iq^I\int F_k d\theta\epsilon &= 0\,,
\end{aligned} \tag{3.7}$$

where $Z = X_I q^I$ denotes the magnetic central charge. We choose as partial supersymmetry breaking conditions

$$\Gamma_{34}\epsilon = i\epsilon\,, \qquad \Gamma_2\epsilon = \epsilon\,. \tag{3.8}$$

This preserves one quarter of the original supersymmetries, i.e., it reduces the number of real supercharges from eight to two.

The integrability conditions following from eqns. (3.7) yield

$$\partial_t\epsilon = \partial_z\epsilon = \partial_\theta\epsilon = \partial_\phi\epsilon = 0\,,$$

$$kZ = \frac{2}{3}e^{W}r(rV' - rW' - 1)\,, \tag{3.9}$$

$$gX^I V_I = -\frac{1}{3}e^{-W}(2V' + W' + r^{-1})\,, \tag{3.10}$$

as well as the charge quantization condition

$$V_I q^I = \frac{1}{3g}\,. \tag{3.11}$$

Notice that (3.11), together with $\Gamma_{34}\epsilon = i\epsilon$, implies the twisting [9]

$$\omega^{34} = 3gV_I A^I\,, \tag{3.12}$$

where ω^{34} is the spin connection on Σ_k.

Using (3.9) and (3.10), the radial equation can be easily solved, to give

$$\epsilon = e^{\frac{1}{2}V}\epsilon_0\,, \tag{3.13}$$

where ϵ_0 denotes a constant spinor subject to the constraints (3.8). When the eqns. (3.9)–(3.11) and (3.13) hold, the Killing spinor equations (3.7) are all satisfied. What remains are the supersymmetry transformations of the gauginos (2.4), which yield

$$\left[-e^{-2W}\frac{k}{r^2}G_{IJ}q^J + \frac{3}{2}e^{-W}\partial_r X_I + 3gV_I\right](\partial_i X^I)\epsilon = 0\,, \tag{3.14}$$

and thus, keeping in mind that $X_I\partial_i X^I = 0$,

$$-e^{-2W}\frac{k}{r^2}G_{IJ}q^J + \frac{3}{2}e^{-W}\partial_r X_I + 3gV_I = \gamma(r)X_I\,, \tag{3.15}$$

where $\gamma(r)$ is some function of r that can be determined by contracting (3.15) with X^I. In this way one obtains

$$\gamma(r) = -\frac{3}{2}e^{-2W}\frac{kZ}{r^2} + 3gX^I V_I\,. \tag{3.16}$$

Using (3.16), (3.9) and (3.10) in (3.15), we finally get the "flow equation" for the scalars,

$$-e^{-2W}\frac{k}{r^2}G_{IJ}q^J + \frac{3}{2}e^{-W}\partial_r X_I + 3gV_I + 3e^{-W}X_I V' = 0\,. \tag{3.17}$$

4. The case $k = 0$: domain walls

Let us first consider the case of a flat manifold Σ_k, i.e. $k = 0$, which will turn out to be completely integrable, with solutions given in terms of Weierstrass elliptic functions. Below we will see that the metric assumes then the form of a domain wall. For $k = 0$ we obtain from eqn. (3.9)

$$V' = W' + r^{-1}\,, \tag{4.1}$$

and hence from (3.10)

$$gX^I V_I = -e^{-W}(W' + r^{-1})\,. \tag{4.2}$$

Using (4.1) in the flow equation (3.17), we get for the scalars X_I

$$\frac{1}{2}e^{-W}X_I' + gV_I + e^{-W}X_I(W' + r^{-1}) = 0\,, \tag{4.3}$$

which is easily solved to give

$$X_I = e^{-2W}r^{-2}\left[-2gV_I\int e^{3W}r^2 dr + C_I\right], \tag{4.4}$$

where the C_I denote integration constants. In what follows it will be useful to introduce the quantities

$$\begin{aligned} a &= C^{IJK}V_IV_JV_K\,, & b &= C^{IJK}V_IV_JC_K\,, \\ c &= C^{IJK}V_IC_JC_K\,, & d &= C^{IJK}C_IC_JC_K\,, \end{aligned} \tag{4.5}$$

with C^{IJK} defined in (A.4). Let us furthermore define the function

$$y(u) = -9a\int e^{3(W+u)}du + \frac{9}{2}g^2 b\,, \tag{4.6}$$

where the new radial coordinate u is given by $u = \ln gr$. Plugging (4.4) into eqn. (A.7), we obtain then the differential equation

$$\dot{y}^2 = 4y^3 - g_2 y - g_3\,, \tag{4.7}$$

where a dot denotes a derivative with respect to u and

$$g_2 = 243g^4(b^2 - ac)\,, \qquad g_3 = \frac{729}{2}g^6(3abc - a^2d - 2b^3)\,. \tag{4.8}$$

The general solution of eqn. (4.7) is given by

$$y = \wp(u + \gamma)\,, \tag{4.9}$$

where $\wp(u)$ denotes the Weierstrass elliptic function, and γ is an integration constant, which we will put equal to zero without loss of generality. g_2 and g_3 are the invariants that are related to the periods ω_1 and ω_2 of the Weierstrass elliptic function by the Eisenstein series

$$g_2 = 60{\sum_{m,n}}'\,\Omega_{m,n}^{-4}\,, \qquad g_3 = 140{\sum_{m,n}}'\,\Omega_{m,n}^{-6}\,, \tag{4.10}$$

where $\Omega_{m,n} = 2m\omega_1 + 2n\omega_2$.

Our supergravity solution is thus given by

$$X_I = \dot{f}^{-2/3}[-2gV_I f + C_I]\,, \tag{4.11}$$

$$ds^2 = \dot{f}^{2/3}[-g^2dt^2 + g^2dz^2 + du^2 + d\Omega_0^2]\,, \tag{4.12}$$

with $f(u)$ defined by

$$f(u) = g^{-3}\int e^{3(W+u)}du = -\frac{y(u)}{9g^3 a} + \frac{b}{2ga}\,. \tag{4.13}$$

Note that also eqn. (4.2), which has not been used above, is satisfied by this solution. From (4.12) we see that the metric is conformally flat like the one of AdS_5, but in contrast to the latter, our solution preserves only part of the supersymmetries, and nonconstant scalar fields are turned on.

Using the expansion

$$\wp(u) = u^{-2} + \mathcal{O}(u^2) \tag{4.14}$$

for $u \to 0$, one sees that asymptotically the solution (4.12) approaches AdS_5 in Poincaré coordinates, and should thus have a dual CFT interpretation. In order to investigate this point further, we expand also the scalars,

$$\ln X_I - \ln X_I^0 = \frac{9}{2} g^2 \left[\frac{C_I}{V_I} a - b\right] u^2 + \mathcal{O}(u^4)\,, \tag{4.15}$$

where $X_I^0 = V_I(2/9a)^{1/3}$. Let us focus for a moment on the STU model (which will be discussed in more detail in section 6). This model can be obtained by compactification of ten-dimensional type IIB supergravity [12], and has two independent scalars with mass squared $m^2 = -4g^2$. They are thus dual to operators of dimension $\Delta = 2$ [1]. For this dimension, the scalars $\ln(X_I/X_I^0)$ behave as $\sim u^2 \ln u$ or $\sim u^2$ for $u \to 0$ [9]. The first kind of behaviour corresponds to the non-normalizable mode associated to the insertion of an operator [13]. From (4.15) we see that in our case the operators dual to the bulk scalars are not inserted. Instead, we have a so-called vev flow,[3] corresponding to a change of vacuum in the dual field theory, in which the operator is given a vev.

Let us now come back to the case of an arbitrary number of vector multiplets, and note that one can cast the solution (4.12) into the domain wall form

$$ds^2 = d\rho^2 + e^{2\omega} ds_4^2\,, \tag{4.16}$$

where the new radial coordinate ρ is defined by $d\rho = \dot{f}^{1/3} du$, $e^{2\omega} = \dot{f}^{2/3}$, and ds_4^2 denotes the flat Minkowski metric in four dimensions. (4.16), together with the scalars (4.11), represent the most general domain wall solution to $D = 5$, $N = 2$ gauged supergravity coupled to vector multiplets. From (4.16) we can compute the holographic $\mathcal{C}$-function of the renormalization group flow, given by [15]

$$\mathcal{C} \propto \left(\frac{d\omega}{d\rho}\right)^{-3}\,, \tag{4.17}$$

which yields in our case

$$\mathcal{C} \propto \frac{\dot{f}^4}{\ddot{f}^3}\,. \tag{4.18}$$

Furthermore, if we use (4.2) as well as $e^\omega = re^W$, we can rewrite the flow equation (4.3) in the form

$$e^\omega \frac{dX_I}{de^\omega} = \beta_I\,, \tag{4.19}$$

[3] See e.g. [14].

where

$$\beta_I = \frac{2(\delta_I{}^J - X_I X^J)V_J}{X^K V_K}\,. \tag{4.20}$$

As the scalars X_I represent coupling constants for the dual operators, and the function e^{ω} determines the physical scale of the dual field theory, we recognize (4.19) as the Callan-Symanzik equation, with the I-th beta function given by (4.20).

Notice that for $k = 0$, supersymmetry is actually enhanced from one quarter to one half, because the constraint $\Gamma_{34}\epsilon = i\epsilon$ in (3.8) can be dropped in this case. The Killing spinors for the domain wall solution (4.16), (4.11) read

$$\epsilon = |\dot{f}|^{1/6}\epsilon_0\,, \tag{4.21}$$

where the constant spinor is subject to the constraint $\Gamma_2\epsilon_0 = \epsilon_0$.

In general, the solution (4.12) has a curvature singularity for $\dot{f} = 0$, e.g. the scalar curvature reads

$$R = \frac{4}{3\dot{f}^{8/3}}[\ddot{f}^2 - 2\dot{f}\dddot{f}]\,. \tag{4.22}$$

In our case, the invariants g_2, g_3 are real, so that $\wp(u)$ is real for $u \in \mathbb{R}$, and one has a real and an imaginary period (although these are not primitive periods for $\Delta \equiv g_2^3 - 27g_3^2 < 0$). This means that if we start from $u = 0$ (AdS region) and go to positive values of u (following the RG trajectory), we will eventually reach a point $u = u_0$ where $\dot{\wp}(u)$ vanishes. The appearance of this curvature singularity is a signal of nontrivial IR physics. According to Gubser's criterion [16], large curvatures in geometries of the form (4.16) are allowed only if the scalar potential is bounded above in the solution.[4] Let us see what this restriction implies here. To this end, we start from the potential in the form (A.9), and use the expression (4.11) for the scalars. This yields

$$V(X) = 6\wp(u)(9a/\dot{\wp}(u))^{2/3}\,. \tag{4.23}$$

As $(9a/\dot{\wp}(u))^{2/3}$ is always positive, and goes to $+\infty$ for $\dot{\wp}(u) \to 0$, we must have $\wp(u_0) < 0$ in order that the potential be bounded above in the solution. It is straightforward to show that this implies

$$\Delta \equiv g_2^3 - 27g_3^2 < 0 \qquad \text{and} \qquad g_3 < 0\,. \tag{4.24}$$

Apart from that, there is another physically admissible solution for $g_2 = g_3 = 0$. In this degenerate case one has $\wp(u) = 1/u^2$, so the metric reduces to that of AdS_5.

It would be interesting to lift a solution with a "bad" singularity to ten dimensions. This is feasible e.g. for the STU model. Probably this leads to pathologies of D3-branes with negative tension and negative charges. We will not attempt to do this here.

A special case appears if the integration constants C_I in (4.4) vanish. One has then from (4.7) after some simple algebra

$$\dot{f}^2 = \frac{2}{3}\ddot{f}f\,. \tag{4.25}$$

[4] It is straightforward to show that the coordinate ρ used in (4.16) assumes a finite value for $u \to u_0$, so Gubser's criterion applies.

In other words, the schwarzian derivative $(2\dddot{f}\dot{f} - 3\ddot{f}^2)/2\dot{f}^2$ vanishes. Using $f = \int e^{3W} r^2 dr$ following from (4.13), the differential equation (4.25) yields

$$e^W r = C(W'r + 1)\,, \tag{4.26}$$

C denoting an integration constant. (4.26) is actually a first integral of the Liouville equation with zero "momentum". To see this, consider the Liouville equation

$$\Delta W = \mu e^{2W}\,, \tag{4.27}$$

where $\mu > 0$ is a constant, in spherical coordinates (r, σ), for Liouville fields W independent of the angular coordinate σ. In this case one has from (4.27)

$$W'' + \frac{W'}{r} = \mu e^{2W}\,. \tag{4.28}$$

The action from which (4.28) follows is invariant under the transformation $r \to \lambda r$, $W \to W - 2\ln\lambda$, with $\lambda \in \mathbb{R}^+$. The first integral associated to this invariance is

$$(W'r + 1)^2 = \mu r^2 e^{2W} - p^2\,, \tag{4.29}$$

where the integration constant p is essentially the Liouville momentum [17, 18]. Comparing with (4.26), we see that W has zero momentum, and corresponds thus to a parabolic Liouville solution [17], given by

$$e^W = \frac{C}{r\ln gr}\,. \tag{4.30}$$

One can now use (4.30) in (4.4) to determine the scalar fields, with the result

$$X_I = gCV_I\,, \tag{4.31}$$

and thus the scalars are forced to be constants in the case $k = 0$, $C_I = 0$.

Determining finally V from (4.1) yields the metric

$$ds^2 = \frac{C^2}{\ln^2 gr}\left[-dt^2 + dz^2 + \frac{dr^2}{r^2} + d\Omega_0^2\right]. \tag{4.32}$$

Introducing the coordinate $u = \ln gr$, one easily sees that (4.32) is AdS_5 in Poincaré coordinates, and thus preserves all supersymmetries.

5. Fixed points of the flow equation

We now come back to the case of general k, and would like to determine the fixed points of the flow equation (3.17), i.e., the points where $\partial_r X_I = 0$, and thus

$$-e^{-2W}\frac{k}{r^2}G_{IJ}q^J + 3gV_I + 3e^{-W}X_I V' = 0\,. \tag{5.1}$$

From (3.9) and (3.10) we get

$$V' = \frac{1}{2r^2}kZe^{-W} - gX^I V_I e^W\,. \tag{5.2}$$

Inserting this into (5.1) yields

$$e^{-2W}\frac{k}{r^2}\left(-G_{IJ}q^J+\frac{3}{2}ZX_I\right)+3g(V_I-X_IX^JV_J)=0\,. \tag{5.3}$$

Obviously we have to distinguish two cases, namely $e^W r$ constant and $e^W r$ not constant. Let us first study the latter case. From the foregoing equation we have then

$$X^I=\frac{q^I}{Z}\,, \qquad X_I=\frac{V_I}{X^JV_J}\,. \tag{5.4}$$

The values $X^I=q^I/Z$ for the scalars are exactly the ones which extremize the magnetic central charge Z.[5] The critical value of Z reads

$$Z=\left(\frac{1}{6}C_{IJK}q^Iq^Jq^K\right)^{1/3}\,. \tag{5.5}$$

Using $q^I=ZX^I$ in the charge quantization condition (3.11), one obtains

$$X^IV_I=\frac{1}{3gZ}\,. \tag{5.6}$$

We will now show that similar to the case $k=0$, the Liouville equation appears also for $k=\pm 1$. To see this, define the field Φ by

$$e^{2\Phi}=e^{2W}+\frac{3kZ^2}{r^2}\,. \tag{5.7}$$

Making use of (3.9), (3.10) and (5.6), it is straightforward to show that

$$(\Phi' r+1)^2=\frac{r^2}{9Z^2}e^{2\Phi}-\frac{k}{3}\,, \tag{5.8}$$

which is again a first integral of the Liouville equation, with momentum given by $p^2=k/3$, as can be seen by comparing with (4.29). The field Φ satisfies thus the Liouville equation

$$\Phi''+\frac{\Phi'}{r}=\frac{1}{9Z^2}e^{2\Phi}\,. \tag{5.9}$$

For $k=1$, we have real momentum, so the solution is hyperbolic [17],

$$e^{2\Phi}=\frac{3Z^2}{r^2\sin^2\left(\frac{\ln gr}{\sqrt{3}}\right)}, \tag{5.10}$$

whereas for $k=-1$ (imaginary momentum), the solution is elliptic,

$$e^{2\Phi}=\frac{3Z^2}{r^2\sinh^2\left(\frac{\ln gr}{\sqrt{3}}\right)}. \tag{5.11}$$

[5]This is analogous to the extremal electric central charge for black holes in the ungauged theory [20].

Determining also V from (5.2), one obtains finally for the metric

$$ds^2 = \frac{k}{\cos(p\ln gr)\sin^2(p\ln gr)}(-dt^2 + dz^2) + \frac{3kZ^2}{r^2}\cot^2(p\ln gr)(dr^2 + r^2 d\Omega_k^2)\,, \qquad (5.12)$$

with the Liouville momentum $p = \sqrt{k/3}$. (5.12) coincides with the solutions found in [7, 8], which are written here in different coordinates that make the connection with Liouville theory more evident.[6] We observe that the line element (5.12) is invariant under the transformations $gr \to 1/gr$ or $gr \to gr\exp(2\pi n/p)$, where $n \in \mathbb{Z}$. In terms of the coordinate $u = \ln gr$ this means $u \to -u$ or $u \to u + 2\pi n/p$. Note that p is real only for $k = 1$. In this case, one encounters a naked curvature singularity for $pu = \pi/2$. For $k = -1$, the singularity is hidden by a horizon, which appears for $u \to \infty$. The solution approaches $AdS_3 \times H^2$ near the horizon. It will be shown below that this geometry has enhanced supersymmetry. Notice finally that the conformal boundary of the solution (5.12) is reached for $u \to 0$.

We come now to the case where $e^W r$ is constant. From (3.1) one sees that this corresponds to a product space $M_3 \times \Sigma_k$, with M_3 a three-manifold to be determined below. Let us decompose q^I and V_I in a part parallel to X^I and a part orthogonal to X^I,

$$q^I = ZX^I + ZP^I{}_J a^J\,, \qquad (5.13)$$
$$V_I = X^J V_J X_I + c_J P^J{}_I\,, \qquad (5.14)$$

where $P^I{}_J = \delta^I{}_J - X^I X_J$ is a projector satisfying $P^I{}_J X^J = X_I P^I{}_J = 0$, and a^I, c_I parametrize the parts orthogonal to X^I. Plugging the above decompositions into (5.1) and eliminating V' by means of (3.9) yields

$$kZ = -gr_0^2 X^I V_I\,, \qquad (5.15)$$
$$d_I = \frac{kZ}{3gr_0^2} G_{IJ} b^J\,, \qquad (5.16)$$

where we defined $r_0 = e^W r$, $b^I = P^I{}_J a^J$ and $d_I = c_J P^J{}_I$. Using eqns. (5.13), (5.14) and (5.16) in (5.15), one gets

$$r_0^2 = kZ^2[G_{IJ} b^I b^J - 3]\,. \qquad (5.17)$$

From (5.17) we conclude that $k = 1$ for $G_{IJ} b^I b^J > 3$, and $k = -1$ for $G_{IJ} b^I b^J < 3$. For $G_{IJ} b^I b^J = 3$ an interesting topological transition occurs.[7] Start e.g. with positive $G_{IJ} b^I b^J - 3$, so that $k = 1$, i.e. Σ_k is a two-sphere. Let then $G_{IJ} b^I b^J - 3$ go to zero, which means that the radius r_0 goes to zero, so that the S^2 shrinks to a point. When $G_{IJ} b^I b^J - 3$ changes sign to become negative, the two-manifold Σ_k restarts to blow up, but now as a hyperbolic space H^2. These topological transitions are similar to the ones considered

[6] It is worth pointing out that also the magnetic branes of Einstein-Maxwell-AdS gravity in arbitrary dimension considered in [19] can be written in terms of functions satisfying the Liouville equation.

[7] In [8] only the case $b^I = 0$ was considered. This leads to $k = -1$.

in [21].[8] Let us examine in more detail the transition point. Using (5.13) and (A.2), one easily shows that $G_{IJ}b^I b^J = 3$ is equivalent to

$$C_{IJK}q^I q^J X^K = 0\,. \tag{5.18}$$

It would be interesting to understand the meaning of this topological transition in the dual conformal field theory.

We still have to determine the function V. Integrating eqn. (3.9) one obtains

$$e^{2V} = (gr)^{3kZ/r_0}\,. \tag{5.19}$$

If we finally introduce the new coordinate ρ defined by $(g\rho)^2 = (gr)^{3kZ/r_0}$, the five-dimensional metric reads

$$ds^2 = (g\rho)^2(-dt^2 + dz^2) + \left(\frac{2r_0^2}{3kZ}\right)^2 \frac{d\rho^2}{\rho^2} + r_0^2 d\Omega_k^2\,, \tag{5.20}$$

so that the manifold is $AdS_3 \times \Sigma_k$.

It turns out that in the case of constant $e^W r$, the constraint $\Gamma_2\epsilon = \epsilon$ on the Killing spinors can be dropped, so that we only impose $\Gamma_{34}\epsilon = i\epsilon$. The above product space solutions preserve thus half of the supersymmetry. The Killing spinors read

$$\epsilon = e^{V/2}P\left[1 + \frac{3kZ}{2r_0^2}(\Gamma_0 t + \Gamma_1 z)\right]\Pi\epsilon_0 + e^{-V/2}(1-P)\Pi\epsilon_0\,, \tag{5.21}$$

where we defined the projectors $P = (1+\Gamma_2)/2$, $\Pi = (1 - i\Gamma_{34})/2$, and ϵ_0 denotes an arbitrary constant spinor.

6. Nonconstant scalars and $k = \pm 1$

We now consider the flow equation (3.17) for the case of nonconstant scalar fields and $k = \pm 1$. To be specific, we consider the STU model which has only one intersection number $C_{123} = 1$ nonzero. This model can be embedded into gauged $N = 4$ and $N = 8$ supergravity as well. Furthermore, it can be obtained by compactification of ten-dimensional type IIB supergravity [12]. This allows to lift the solutions presented below to ten dimensions.

The prepotential reads

$$\mathcal{V} = STU = 1\,. \tag{6.1}$$

Taking $S = X^1$, $T = X^2$ and $U = X^3$ one gets for the matrix G^{IJ}

$$G^{IJ} = 2\begin{pmatrix} S^2 & & \\ & T^2 & \\ & & U^2 \end{pmatrix}. \tag{6.2}$$

[8] Note however that in [21] cycles in the internal Calabi-Yau manifold shrink to zero and then blow up into a different Calabi-Yau, whereas in our case the moduli X^I do not go to zero at the transition.

Without loss of generality we assume $V_I = 1/3$. Considering S as the dependent field (i.e. $S = 1/TU$) we find for the potential

$$V(X) = 2\left(\frac{1}{U} + \frac{1}{T} + TU\right), \tag{6.3}$$

which has a minimum $V_{\min}(U = T = 1) = 6$.

The equations to solve are given in appendix B. Maldacena and Nuñez [9] found solutions to these equations with nonconstant scalars for the special case $T = U$ and $q^2 = q^3 = 0$, where the operator dual to $\ln T$ is inserted. We will present here a different solution, where the operators dual to the bulk scalars are not inserted, but instead they are given a vev by a change of vacuum in the dual field theory.

In appendix B it is shown that for the STU model, a special solution to the eqns. (3.9)–(3.11) and (3.17) is given by

$$\begin{aligned}
q^1 &= q^2 = q^3 = \frac{1}{3g}\,,\\
ds^2 &= e^{\frac{k}{6}u^2}u^{-2}\left[1-\frac{k}{6}u^2\right]^{-2/3}(-dt^2+dz^2) + g^{-2}u^{-2}\left[1-\frac{k}{6}u^2\right]^{4/3}(du^2+d\Omega_k^2)\,,\\
S &= U = \left[1-\frac{k}{6}u^2\right]^{1/3}\,,\\
T &= S^{-2}\,,
\end{aligned} \tag{6.4}$$

where again $u = \ln gr$. The conformal boundary is approached for $u \to 0$. As in (4.15) we can expand the moduli for $u \to 0$, and see that they behave as the normalizable mode, so that the dual operators are not inserted. Note that if $k = 1$, the above metric becomes singular for $u^2 = 6$. In order to decide whether this singularity is allowed or not, one cannot use the criterion of [16], because strictly speaking this applies only to geometries of the domain wall form. We can however lift our solution to ten dimensions using the rules given in [12], and then use the criterion of [9] which states that the component g_{00} of the ten-dimensional metric should not increase as we approach the singularity. In our case it is easy to show that g_{00} blows up like $(1 - u^2/6)^{-1}$, so that the singularity would not be allowed according to [9].

7. Electric solutions

One can try to find an electric analogue of (3.1). An obvious ansatz would be

$$ds^2 = e^{2V}(dx^2 + dy^2) + e^{2W}(dr^2 + r^2 d\sigma_k^2)\,, \tag{7.1}$$

where again V and W are functions of r only, and $d\sigma_k^2$ denotes the standard metric on two-dimensional Minkowski space ($k = 0$), de Sitter space dS_2 ($k = 1$) or anti-de Sitter space AdS_2 ($k = -1$). A possible choice is

$$d\sigma_k^2 = -dt^2 + f_k^2(t)dz^2\,, \tag{7.2}$$

with

$$f_k(\theta) = \begin{cases} 1\,, & k=0 \\ \sinh t\,, & k=1 \\ \sin t\,, & k=-1\,. \end{cases} \tag{7.3}$$

For the gauge fields we take

$$F^I_{tz} = kq^I f_k(t)\,, \qquad A^I_z = kq^I \int f_k(t)dt\,. \tag{7.4}$$

Using projection conditions on the Killing spinors analogous to (3.8), one finds that the charge quantization condition is now given by

$$V_I q^I = \frac{1}{3ig}\,, \tag{7.5}$$

so that either the charges q^I or the coupling constant g have to be imaginary, which is of course unphysical. Nevertheless the possibility of having imaginary coupling constant g suggests that supersymmetric electric solutions of the type considered here might exist in the exotic de Sitter supergravity theories introduced by Hull [22].

8. Final remarks

We conclude this paper by pointing out some possible extensions of the work presented here. First of all, one notes that for the magnetic strings, the solutions with parabolic, hyperbolic or elliptic monodromy (cf. eqns. (4.30), (5.10) and (5.11)) essentially coincide with the Weierstrass elliptic function $\wp(u)$ (with $u = \ln gr$) for the degenerate case where the discriminant $\Delta = g_2^3 - 27g_3^2$ vanishes (see e.g. [23]). This suggests that there might exist more general magnetically charged string solutions, whose metric is given in terms of elliptic functions. If these solutions preserve some supersymmetry, it is clear from the results of our paper that nonconstant scalars have to be turned on in this case.

Furthermore, it would be interesting to find the nonextremal or rotating generalizations[9] of the supersymmetric strings found here. Holographically, this would correspond to considering respectively field theories at finite temperature or on a rotating manifold.

We hope to report on these points in a future publication.

Acknowledgments

This work was partially supported by INFN, MURST and by the European Commission RTN program HPRN-CT-2000-00131, in which S.L.C. and D.K. are associated to the University of Torino. The authors would like to thank J. Maldacena and W. Mück for clarifying correspondence.

[9]In four dimensions, the rotating generalization of the magnetically charged, one quarter supersymmetric soliton was found in [4].

A. Useful relations in very special geometry

We list here some useful relations that can be proven using the techniques of very special geometry:

$$\partial_i X_I = -\frac{2}{3}G_{IJ}\partial_i X^J\,, \qquad X_I = \frac{2}{3}G_{IJ}X^J \tag{A.1}$$

$$G_{IJ} = \frac{9}{2}X_I X_J - \frac{1}{2}C_{IJK}X^K\,, \tag{A.2}$$

$$G^{IJ} = 2X^I X^J - 6C^{IJK}X_K\,, \tag{A.3}$$

where the C^{IJK} are defined by

$$C^{IJK} = \delta^{II'}\delta^{JJ'}\delta^{KK'}C_{I'J'K'}\,. \tag{A.4}$$

Eqn. (A.3) can be shown using the "adjoint identity"

$$C_{IJK}C_{J'(LM}C_{PQ)K'}\delta^{JJ'}\delta^{KK'} = \frac{4}{3}\delta_{I(L}C_{MPQ)} \tag{A.5}$$

of the associated Jordan algebra [24]. Using (A.3) one obtains furthermore

$$X^I = \frac{9}{2}C^{IJK}X_J X_K\,, \tag{A.6}$$

and

$$\frac{9}{2}C^{IJK}X_I X_J X_K = 1\,. \tag{A.7}$$

The scalar potential (2.2) can also be written as [5]

$$V(X) = 9V_I V_J\left(X^I X^J - \frac{1}{2}G^{IJ}\right). \tag{A.8}$$

Using (A.3), this can be cast into the form

$$V(X) = 27C^{IJK}V_I V_J X_K\,. \tag{A.9}$$

B. Solutions with nonconstant scalars for $k = \pm 1$

For the STU model considered in section 6, we introduce the rescaled fields $x^I = re^W X^I$ so that $x^1 x^2 x^3 = r^3 e^{3W}$. The equations (3.9)–(3.11), and (3.17) become

$$g\sum_I q^I = 1\,, \tag{B.1}$$

$$g\sum_I x^I = -\left(2\dot{V} + \dot{W} + 1\right), \tag{B.2}$$

$$k\sum_I \frac{q^I}{x^I} = -2\left(\dot{W} + 1 - \dot{V}\right), \tag{B.3}$$

$$-kq^I - \dot{x}^I + 2gx^{I2} + (2\dot{V} + \dot{W} + 1)x^I = 0\,, \tag{B.4}$$

where the dot denotes a derivative with respect to $u = \ln gr$. Using (B.2) in (B.4) we find

$$\begin{aligned} kq^1 + \dot{x}^1 + g(-x^1 + x^2 + x^3)x^1 &= 0\,, \\ kq^2 + \dot{x}^2 + g(x^1 - x^2 + x^3)x^2 &= 0\,, \\ kq^3 + \dot{x}^3 + g(x^1 + x^2 - x^3)x^3 &= 0\,. \end{aligned} \tag{B.5}$$

Note that from $x^1x^2x^3 = r^3 e^{3W}$ we derive

$$3(\dot{W} + 1) = \sum_I \frac{\dot{x}^I}{x^I}\,, \tag{B.6}$$

which confronted with the sum of (B.2) and (B.3) gives the consistency condition

$$\sum_I \frac{\dot{x}^I}{x^I} = -g\sum_I x^I - k\sum_I \frac{q^I}{x^I}\,, \tag{B.7}$$

which is satisfied by the system (B.5).

One can then solve the eqns. (B.5) and use $x^1x^2x^3 = r^3e^{3W}$ to find W and finally (B.2) or (B.3) to determine V so that

$$ds^2 = \left(\prod_I x^I\right)^{-1/3} e^{-g\int\sum_J x^J du}\left(-dt^2 + dz^2\right) + \left(\prod_I x^I\right)^{2/3}\left(du^2 + d\Omega_k^2\right). \tag{B.8}$$

If we introduce the functions

$$\begin{aligned} y^1 &= x^1 + x^2 - x^3\,, \\ y^2 &= x^1 - x^2 - x^3\,, \\ y^3 &= x^1 - x^2 + x^3\,, \end{aligned}$$

and the constants

$$\begin{aligned} Q^1 &= -k\left(q^1 + q^2 - q^3\right), \\ Q^2 &= -k\left(q^1 - q^2 - q^3\right), \\ Q^3 &= -k\left(q^1 - q^2 + q^3\right), \end{aligned}$$

the system (B.5) becomes

$$Q^1 - \dot{y}^1 + gy^2y^3 = 0\,, \tag{B.9}$$

$$Q^2 - \dot{y}^2 + gy^1y^3 = 0\,, \tag{B.10}$$

$$Q^3 - \dot{y}^3 + gy^1y^2 = 0\,. \tag{B.11}$$

From (B.9) and (B.10) one finds

$$\begin{aligned} y^1 + y^2 &= e^{g\int y^3du}\left[(Q^1 + Q^2)\int e^{-g\int y^3du}du + K_+\right], \\ y^1 - y^2 &= e^{-g\int y^3du}\left[(Q^1 - Q^2)\int e^{g\int y^3du}du + K_-\right], \end{aligned}$$

where $K_\pm$ denote integration constants. Using these expressions in (B.11) one obtains an integro-differential equation for y^3 which is quite complicated. However a solution can be found e.g. in the simple case $Q^1 + Q^2 = 0$ and $K_+ = 0$. This corresponds to

$$q^1 = q^3 \,, \qquad q^2 = \frac{1}{g} - 2q^1 \,.$$

If we introduce the function

$$h = Q^1 \int e^{g \int y^3 du} du + \frac{K_-}{2} \,,$$

we obtain

$$y^1 = -y^2 = Q^1 \frac{h}{\dot{h}} \,.$$

Eqn. (B.11) becomes

$$\partial_u^2 \ln \dot{h} = gQ^3 - \left(\frac{gQ^1 h}{\dot{h}} \right)^2 \,. \tag{B.12}$$

A special solution can be found using the ansatz

$$h(u) = h_0 e^{\alpha u^2} \,, \tag{B.13}$$

which solves (B.12) if

$$\alpha = \frac{g}{2} Q^3 \,, \qquad Q^1 = \pm Q^3 \,.$$

Choosing the plus[10] we get the solution (6.4).

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JHEP03(2003)023